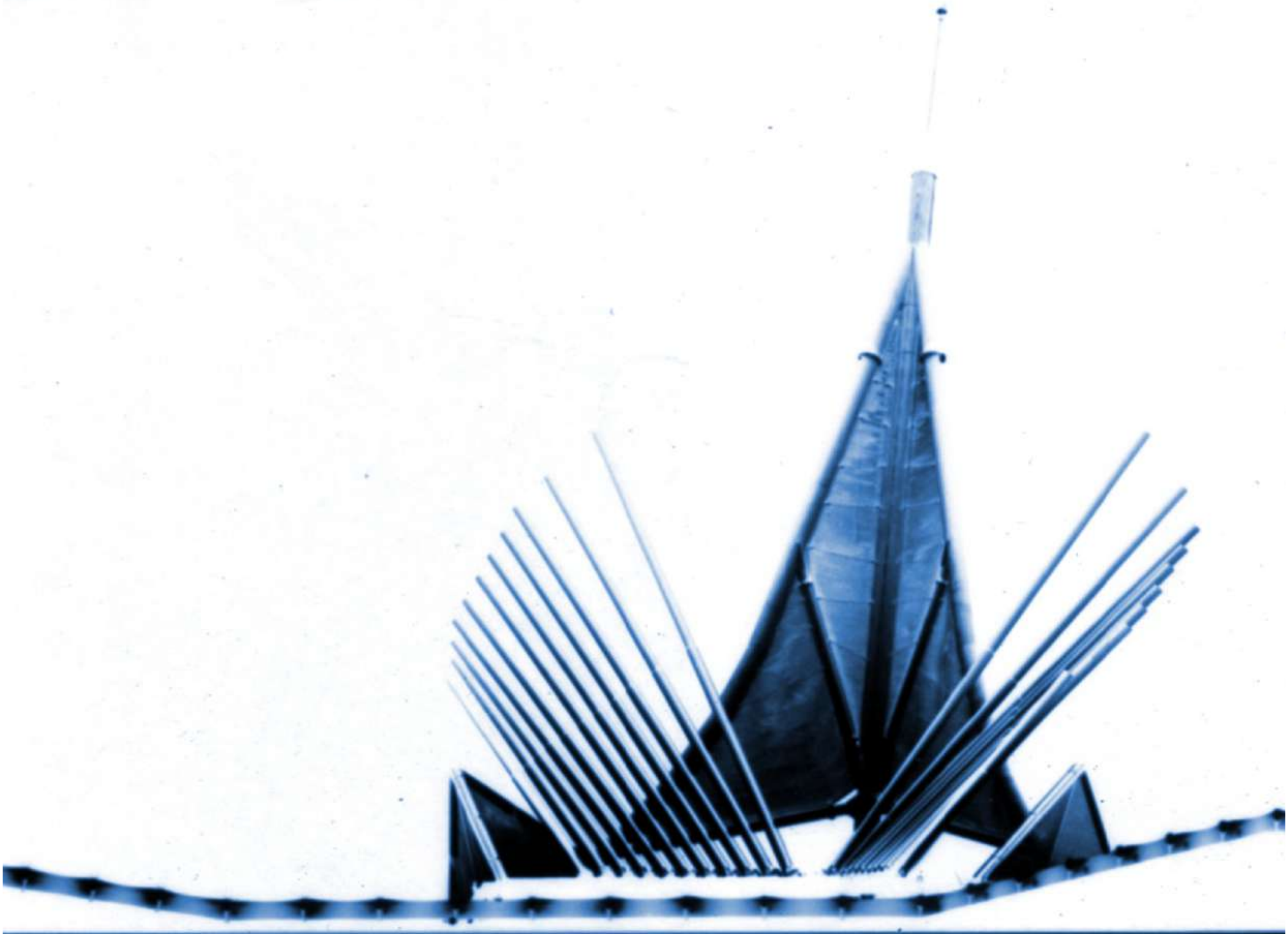
Pre-Proceedings Of The Twenty-Sixth ICMI Study

Advances In Geometry Education



Reims, France,
INSPÉ de l'académie de Reims,
University of Reims Champagne-Ardenne
April 23 – 26, 2024

The Twenty-Sixth ICMI Study

Advances in Geometry Education

CONFERENCE PRE-PROCEEDINGS

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The Discusion Document

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THE NOTION OF ANGLE AND THE GGBOT AS A TOOL-TO-THINK-WITH... OR WITHOUT

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This study is about how geometrical reasoning can be supported through experiences with artifacts. Specifically, taking a semiotic perspective, we focus on the potential of primary school activities exploiting a drawing robot, the GGBot, a descendant of Papert's robotic turtle, in relation to the concept of angle. We analyze the interview of a 5th grader who participated in such activities to highlight his way of seeing the contour of a figure, that, we claim, showcases the extent to which GGBot's functionalities can become a "tool-to-think-with" (or, actually, without).

INTRODUCTION

Managing the theoretical nature of mathematical objects and the spatial nature of their (physical or mental) representations is a complex process with a long development (e.g., Fischbein, 1993). Geometrical reasoning is strongly connected with developing various ways of seeing and managing representations of geometrical objects, also making use of analytical reasoning (Duval, 1995). Here we will consider analytical reasoning as a specific kind of geometrical reasoning, that is related to recognizing subunits and composing and decomposing representations of figures (Duval, 1995).

It is widely recognized that the use of artifacts can provide experiences promoting multiple and dynamic representations of geometrical objects, that with appropriate tasks can be highly effective in fostering analytical and geometrical reasoning (e.g., Leung et al., 2023). This study provides an example of how during such experiences the artifact can support human cognition, becoming a tool-to-think-with, and it can proceed to become internalized, giving birth to a psychological tool (Vygotsky, 1978). More specifically, we focus on the geometrical notion of angle, because it is crucial in primary school geometry and it is one of the most challenging and misleading geometrical objects (e.g., Fischbein, 1993; Devichi & Munier, 2013), although it is experienced quite early in school (e.g., Bartolini Bussi & Baccaglini-Frank, 2015). Indeed, such notion is epistemologically challenging: for example, as described by Freudenthal (1973), when considering a pair of half lines with a common origin, in *Elementary Geometry* an angle is viewed as a static part of a plane comprised between such an unordered pair, while in *Goniometry* it is viewed as a dynamic turn between an ordered pair.

The protagonist of our study is Sam, a 5th grade student who learned about angles in grades 4 and 5 through a sequence of activities with GGBot, a descendant of Papert's drawing turtle (Baccaglini-Frank & Mariotti, 2022) that we will introduce in the following sections. Here we analyze part of a task-based interview about a polygon and its angles. The analyses focus on the signs Sam produced through geometrical and analytical reasoning. We will argue that such results showcase the extent to which GGBot's functionalities can become a "tool-to-think-with" (or, actually, without).

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THEORETICAL FRAMEWORK AND RESEARCH QUESTION

This study is grounded in *Theory of Semiotic Mediation* (Bartolini Bussi & Mariotti, 2008), a Vygotskian socio-constructivist theory, according to which students are guided by their teacher to construct mathematical knowledge by solving appropriately designed tasks with the use of appropriately chosen artifacts – that is, artifacts and mathematical tasks that are considered helpful for unfolding the target mathematical meanings. During a first phase of the *didactic cycle* students use the *artifact* (e.g., a physical compass, the dragging tool in a Dynamic Geometry Environment) to solve a task and develop *utilization schemes*. The artifact together with such schemes constitutes an *instrument* in the sense of Rabardel (1995), through which the students produce *artifact signs*, closely related to the task and to the artifact. We can now look at two developmental processes that the activity of working with an artifact can foster.

One process concerns the unfolding of the artifact's *semiotic potential*, the artifact's double relationship with personal meanings (related to artifact signs) and with mathematical meanings (related to *mathematical signs*), that the teacher aims to construct through the orchestration of mathematical discussions during the didactic cycles.

A second process concerns the student's *internalization* (e.g., Bartolini Bussi & Mariotti, 2008), the "transformation" of what has been a mainly external process into an internal process. At this phase the student will produce artifact signs and possibly make references to the artifact, even in absence of the artifact. Moreover, this process can go a step further: the instrument can be used as a *psychological tool* (Kozulin, 1998) to reason about and solve a problem. At this phase it is possible to observe the emergence of artifact signs, and possibly mathematical signs, without any observable physical action or presence of the artifact. In other words, it will have become a tool-to-think-with(out).

The semiotic potential of the GGBot with respect to the notion of angle

How does this apply to our study? Our artifact is the GGBot (short for "GREATGeometryBot"), an object moving on wheels (see Figure 1), similar to Papert's turtle, that can be programmed using a graphical block-based coding language similar to Scratch (it is realized in SNAP!). The two back wheels and a metal sphere in the front allow it to move on the plane (Figure 1).

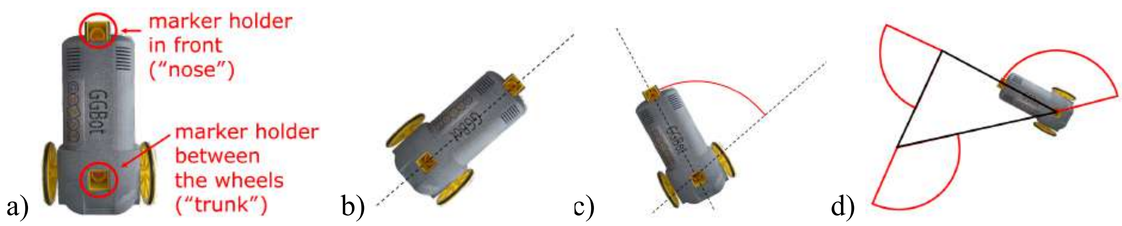


Figure 1: a) top view of GGBot with nose and trunk marker holders; b) translation movement direction; c) direction of departure and arrival after a rotation with vertex at the "trunk" marker, and trace mark left by a red marker in the "nose"; d) trace marks left if the GGBot is programmed to "do the path" of the black triangle.

GGBot has two marker holders, one in front ("nose") and one between the wheels ("trunk") so that it can "provide situated signs that can be elaborated into geometrical notions – such as segment, vertex, angle, rotation, polygon – while still carrying the situatedness given by the real movement of the physical artefact" (Baccaglini-Frank & Mariotti, 2022, p. 2660). The possible movements (translations and rotations) and trace marks are shown in Figure 1.

GGBot's semiotic potential with respect to the notion of angle consists in the relationships between the signs it can produce as it "turns changing direction" and seeing an angle as the rotation that takes one side of the angle onto the other (see Figures 1b and 1c). Concerning angles of polygons, GGBot's greatest semiotic potential is respect to "exterior angle(s)", seen as the change(s) in direction of a person walking along the contour of the figure (Figure 1d).

Such potential can only be realized through well-designed tasks. A dual pair of tasks that were used frequently during the didactic cycle in Sam's class, were the following: the *planning task*, or *figure-to-code task* (students are given the name of a figure and asked to produce a code with the blocks, working in pairs, to make GGBot draw the required figure on the paper); the *prediction task*, or *code-to-figure task* (students are given a code and asked to predict the trace mark GGBot will leave on the paper when executing such a code).

As Baccaglini-Frank & Mariotti (2022) have highlighted,

> "an essential feature of the semiotic potential of this artefact is its building on the relationship between the global movement and its breaking up into steps and turning points and the geometrical meaning of a polygon at a global and an analytical level. From a cognitive point of view […], the task consists in breaking down a path that is imagined to be generated through physical continuous motion, into geometrical elements […] of a different nature: they are static and discrete." (p. 2662).

Specifically, concerning the notion of angle, the artifact GGBot can support the development of geometrical reasoning stemming from a "temporal" and dynamic conception, similar to the one described by Freudenthal in his Goniometry. Moreover, the dual tasks described above have the potential of eliciting analytical reasoning, since in both cases a "whole" figure needs to be decomposed into (or recomposed from) smaller components: in the case of the angle these are single (parts of) sides, with a vertex in common and with a specific reciprocal inclination, given by "how much" GGBot needs to turn to face first one direction and then the other.

Research questions

Our questions are grounded in the theoretical framework above, and we ask specifically: Is it possible for GGBot to act as a psychological tool for solving primary school geometry tasks? If so, how can such a tool support students' analytical (recognizing components and using them to compose/decompose a figure) and geometric reasoning (recognizing and using specific representations of) concerning angle?

METHODS

We analyze here part of a task-based interview with Sam, a 5th grade student, whose class had worked with GGBot since the beginning of 4th grade, following the proposals of the *PerContare* project (teacher guides designed by many researchers including the first two authors, see Bartolini Bussi & Baccaglini-Frank, 2015; Baccaglini-Frank et al., 2023). Initially the activities only made use of the marker in the trunk and of right-angle rotations, for both planning tasks and prediction tasks; subsequently, also the marker in the nose was inserted, and through open problems the students were encouraged to discover new commands with parameters for controlling the amplitude of the rotations (turns) and the length of the translations (steps).

Sam's interview was part of a larger data collection aimed at assessing students' learning at the end of a 3-year cycle of teachers' use of the *PerContare* materials. After assigning and collecting a written set of tasks to 112 5th graders in 5 classes, two researchers (the second author is one of them) visited each classroom and individually interviewed selected students in a quiet room outside the students' regular classroom. Sam was one of the students selected because of the mathematical depth of some of their answers; others were selected because of their mistakes or reference to specific artifacts that helped them think. During the interview, the students were shown their previous written work and asked to further explain their thinking. Moreover, the researchers would ask additional questions when they noted clues of interiorization of a particular artifact. This was very clear in the case of Sam. Figure 2 shows: (2a) the original question ("How many internal angles does this figure have?", followed by the request to explain own reasoning; (2b) Sam's first and (2c) second drawings in response to researchers' questions.

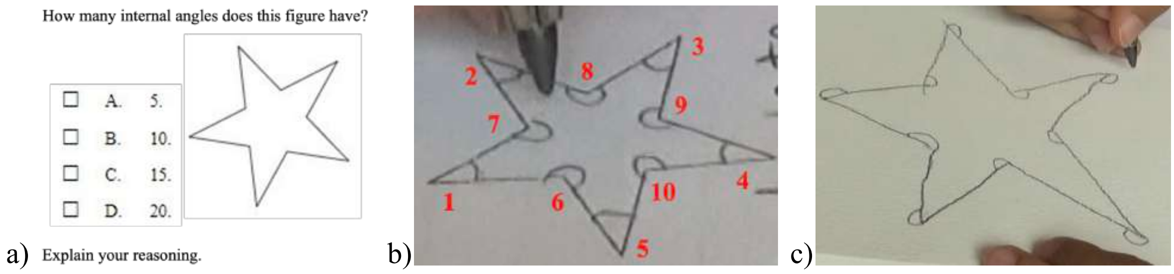


Figure 2: a) original task; b) first figure Sam used to explain his reasoning with numbers indicating the order of his counting; c) Sam's second figure, showing what "the nose would draw"

We selected this task and Sam's answers corresponding to figures 2b and 2c because of Sam's explanation of the relationship between angles drawn with markers in the trunk and in the nose of GGBot, and because the original question introduces the slippery term "internal angles" that frequently induces incorrect answers (this was an item of Italian national assessment test for grade 5 in 2009: in the national student sample, the percentage of A answers was 66%). GGBot was not available in the physical or virtual form during the interview.

To perform the analyses we did the following. We transcribed the interview and searched in the text and video for artifact signs and mathematical signs (verbal utterances, written signs on the paper used, gestures). Since the GGBot was never available during the interview, we interpreted the artifact signs as hints that the internalization of GGBot as a psychological tool. We then related such artifact signs to the mathematical signs produced by Sam, and we proceeded to identify instances of analytical thinking (when the signs are used to distinguish components or possible decompositions or re-compositions of angle) and geometrical thinking (more in general, when the signs are used to talk and reason about representations of angle), till we reached agreement.

ANALYSIS

We selected 3 excerpts that are particularly illuminating with respect to Sam's internalization of the GGBot relative to the notion of angle. We provide summaries of skipped parts around each excerpt.

Excerpt 1: How would GGBot do it?

In the written task, asking for the number of interior angles in the star (Fig. 2a), Sam had answered "10" and explained on the sheet of paper: "I calculated all the internal angles (without falling in the trap)”. When one of the interviewers asked to say something more about the “trap”, he explained the way he counted the angles producing the Figure 2b. Referring to such angles, to better explain how he reasoned, Sam mentioned “turns” (Italian: “svolte”): the interviewers immediately asked him more about what these referred to, and he spoke of the movements of GGBot (original excerpt here).

Sam: So let’s assume it starts from here: it has to go straight and turn, so this is already an angle. Then it has to go straight and turn, and it’s 2 angles. [He repeats the refrain “go straight and turn” four times] Straight, turn, 7. Then it has to go straight again and then turn, and it’s 8. It has to come, uh, it has to go straight and then turn, that's 9. Go straight and turn, and it’s 10… and then come back to where it started from.

Table 1 shows correspondences between GGBot’s movements, Sam’s verbal utterances and his gestures associated to the figural units he identifies in the star (segments and angles).

Movement	Verbal utterances	Hand gestures
Step forward	“go straight” or only “straight”	He sweeps a segment with the right index finger
Rotation	“turn”	He points the index finger at a vertex and rotates it; after counting angle number 5, the finger rotation is less emphatic

Table 1: Overview of GGBot’s predicated movements, Sam’s verbal utterances and his gestures

Sam temporalizes the figure by looking at its contour as a path to be followed – in this attempt we see an instance of internalization: Sam talks as if an imagined GGBot is tracing the star. Internalization is also evident in the turning gestures: they change during the excerpt, becoming less emphasized as if the movement is just imagined or unessential. The small arcs Sam had previously drawn (see Figure 2b) are different from those that the GGBot would draw, but this confirms our point: internalization is more than a replication of a past experience. Rather, Sam uses GGBot’s movement as a (psychological) tool for supporting analytical reasoning and detecting angles in the spatial and dynamic essence of its figural units: just a turn preceded and followed by a straight path.

Excerpt 2: What about the GGBot’s nose?

The interviewers, curious about the marks that Sam uses to count the times GGBot turns, and so the angles of the star, and their correspondence to marks that either of GGBot’s markers would leave, ask Sam what GGBot would draw if it had markers both in the trunk and in the nose (original excerpt here). Sam produced Figure 2c and the signs analyzed in Table 2.

Again we can see how GGBot supports analytical reasoning to decompose the path into a combination of translations and rotations, but there is more. The predicted movements of the GGBot’s nose are used as psychological means to solve a new task and not just to reproduce a known procedure. Indeed, the deixis (see “it makes *this*” when Sam is drawing the arc, Table 2) manifests a deeper identification between Sam and the robot: Sam’s gestures are no longer just a reference to the movement of the GGBot, but gestures and GGbot’s motion overlap intimately. Sam talks as if the hand holding the pencil *was* the GGBot. Moreover, Sam does not trace the actual angle that GGBot “makes with the nose”, but his verbal and gestural productions suggest that it is just imagined. When Sam claims that GGBot “lies here on the tip”, the gesture points to the lengthening (just mimicked) of a side (see

Figure 4) which corresponds to the direction taken by a fictional GGBot. In absence of a physical artefact, Sam's gestures are tools that realize the body syntonicity with the imagined GGBot.

Predicted nose trace mark	Verbal utterances	Pencil gestures
Figure 3 	"go straight" or only "straight"	The pencil runs along one side
Figure 4 	"it makes this" or "it lies here on the tip and it makes this with the nose"	The pencil draws an arc from this vertex to an inner point on the consecutive side; it moves back and forth along the side around the vertex, and then along the drawn arc.

Table 2: Predicated nose trace mark, Sam's verbal utterances, and pencil gestures from excerpt 2

Excerpt 3 – Comparing different signs for angles

Intrigued, the researchers ask Sam to better explain the difference between the angles he drew in Figures 2b and 2c (original excerpt here). He explains:

Sam: Because GGBot instead of making the angles as they are on the protractor, it works with the supplementary angles, if I am not mistaken, which are on the basis of 180. Because if we made a 90-degree angle [he draws a right angle]… oh god, maybe with 90 you don't understand so much....

and, invited by a researcher to show a drawing, he produces the signs in Table 3.

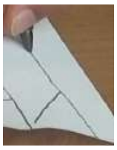
Predicted trace marks	Verbal utterances	Pencil gestures
Figure 5 	"an obtuse [acute] angle"	He draws an acute angle
Figure 6 	"instead of going straight", "it will go here on the brink"	He traces the extension of one side of the angle; points at the acute angle; runs along one of its sides and stops at its vertex
Figure 7 	"with the nose, to turn, it will do this", "on the basis of the supplementary angles", "let's say this is 30 […] and this 50 [degrees]"	He draws an arc from the vertex of the acute angle to its other side; then he draws more lightly also an arc for the obtuse angle

Table 3: Predicated nose trace mark, Sam's verbal utterances, and pencil gestures from excerpt 3

Sam is now using formal mathematical signs and the GGBot's movement to focus on the features of different representations of angle, demonstrating sophisticated geometrical reasoning. He speaks of "supplementary angles" to distinguish angles made by GGBot from those of the protractor (another artifact that he had the opportunity to experience). Interestingly, he uses dynamic expressions (*do, make, works*) to indicate movements when referring to the former, and static ones (*are on*) when speaking about the latter. His control over the mathematical concept of supplementary angles seems quite strong: he starts with an example (right angle) but quickly changes his mind, acknowledging that in such a special case, his example would have been less understandable, since both the exterior angle and the interior one would have both been right angles. Sam then continues to construct his example, mixing artifact signs and mathematical signs (Table 3): the verbal utterances about GGBot's movement are intertwined with more general and atemporal forms ("obtuse angle", "on the basis of the supplementary angles", "let's say this is 30"). Moreover, this example allows him to better explain what he was referring to in excerpt 2 (Table 2), since after that he re-draws the arc associated to the action of "turning" starting from a side of the angle, not its vertex: this is coherent with the "nose" marker's trace, i.e. with the actual angle of rotation of the GGBot.

Shortly after, Sam identifies a pair of supplementary angles, and he says:

> Sam: …ah no, sorry. This one is 30 and this one is 150. He is turning, he makes an angle of 150 degrees [he retraces the small arc]. Internally there is 30 [he retraces the small arc] and together they make 180 [he marks the two horizontal sides a whole]. Because maybe we see the initial angle like this [he retraces the sides of the acute angle], but the angle he sees is this [he retraces the sides of the obtuse angle].

Sam talks about the two angles drawn earlier (Figure 7) by mixing different signs: the obtuse angle is drawn turning (It: "svoltando"*)*, while the acute angle *is* inside. The verbal expression "there is" addresses a static dimension that is less related to the process of drawing the angle, while the gesture of retracing the small arc in the acute angle recalls the trace mark GGBot would make when rotating. Possibly, these different takes on angles mirror two way of looking at – and reasoning about – angles: one way is static in nature – angles *are somewhere* and, for example, "they are on the protractor" – and the other is dynamic and recalls the angle that the GGBot "makes turning". The dynamism is partially overcome by the whole figure representing supplementary angles that Sam now sees as two adjacent angles, as the gesture reveals. In the last utterance the dynamism has been completely overcome, as if the original angle that "we see" and the one that "he [GGBot] sees" have blended together, giving rise to something new, indicated through a single repeated gesture with the pencil along their sides. This interpretation is strengthened by Sam's use of "see", instead of "do".

CONCLUSION AND DISCUSSION

We set out to explore whether GGBot could act as a psychological tool solving primary school geometry tasks, and found compelling evidence that, yes, it is indeed possible. The case of Sam allowed us to gain insight into how such a tool supported his analytical and geometric reasoning on a specific task. Digging deeper into what Sam had internalized, we discovered how GGBot had supported his conceptualization of the notion of angle, and in particular of supplementary angles, proving that it plays the role of tool to recall and productively use geometrical notions, developing ever more effective objects-to-think-with (or, in our case, without).

Our analyses revealed how working with GGBot led Sam to re-thinking a figure, decomposing it and using analytic reasoning to relate the commands, movements, trace marks and figural units in a global whole. This is possible not only through the use of GGBot, but thanks to the dyad composed of the artifact and the dual tasks we have designed, which elicit processes of (de)composition and anticipation. Specifically, we discovered that the sequence of activities with GGBot that was proposed to Sam led him to conceive angles as rotations, but also as geometrical figures comprising two sides and an "inclination" between them, recognizing different kinds of angles (right, acute, obtuse), their mutual relationships (two supplementary angles) and angles of closed figures (internal and external). In particular, Sam constructed a theoretical lens that let him interpret the expression "internal angles" in the initial task coherently with formal geometry and correctly solve it, differently from many other Italian students in the national assessment. Moreover, thanks to synergies with other artifacts such as the protractor, Sam merges dynamic and static experiences, reaching a generalized static and atemporal (but with dynamic features encapsulated within) conception of angle.

In this perspective, the learning path through the *PerContare* activities with GGBot fully exploited the artifact's semiotic potential, leading Sam to develop a psychological tool that he can use in different ways, unpacking as necessary the interiorized experiences when necessary, for example, to "see" a geometric figure as a part of the plane enclosed by consecutive segments, or supplementary angles as the "pair" of angles, one "seen from the figure" and the other "seen by GGBot" as it traces the contour. Of course there remain many open questions and issues to study in more depth. Just to mention one in which we are particularly interested, we are curious to explore how the dual tasks presented can be harbingers of similar activities in dynamic geometry (with step-by-step construction open problems) useful for enhancing key geometric skills (Leung et al., 2023).

References

Baccaglini-Frank, A., Funghi, S., Maracci, M., & Ramploud, A. (2023). Learning about multiplication by comparing algorithms: "One times one, but actually they are ten times ten". *The Journal of Mathematical Behavior, 70*, 101024.

Baccaglini-Frank, A., & Mariotti, M. A. (2022). "Doing well" in the Teaching for Robust Understanding approach revealed by the lens of the semiotic potential of tasks with the GGBot. In *Proc. of the 12th Congress of the European Society for Research in Mathematics Education (CERME12)* (pp. 2658–2665). ERME / Free University of Bozen-Bolzano.

Bartolini Bussi, M. G., & Baccaglini-Frank, A. (2015). Geometry in early years: sowing seeds for a mathematical definition of squares and rectangles. *ZDM*, *47*(3), 391–405.

Bartolini Bussi, M. G., & Mariotti, M. A. (2008). Semiotic Mediation in the Mathematics Classroom: Artefacts and Signs after a Vygotskian Perspective. In *Handbook of International research in Mathematics education* (2nd edition) (pp. 746–783). Routledge Taylor & Francis Group.

Devichi, C., & Munier, V. (2013). About the concept of angle in elementary school: Misconceptions and teaching sequences. *The Journal of Mathematical Behavior*, *32*(1), 1–19.

Duval, R. (1995). Geometrical Pictures: Kinds of representation and specific processing. In Sutherland R. & Mason J. (Eds.), *Exploiting mental imagery with computers in mathematics education* (pp. 142–157). Springer.

Fischbein, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics*, *24*(2), 139–162.

Freudenthal, H. (1973). *Mathematics as an educational task*. Reidel.

Kozulin, A. (1998). *Psychological tools: a sociocultural approach to education*. Harvard University Press.

Leung, A., Baccaglini-Frank, A., Mariotti, M.A., Miragliotta, E. (2023). Enhancing geometric skills with digital technology: the case of dynamic geometry. In B. Pepin, G. Gueudet, & J. Choppin (Eds.), *Handbook of digital resources in mathematics education*. Springer.

Rabardel, P. (1995). *Les hommes et les technologies – Approche cognitive des instruments contemporains.* Armand Colin.

Vygotsky, L. S. (1978). *Mind in society: the development of higher psychological processes*. Harvard University Press.